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Comparative Analysis of Fourier Series Analysis and Holtwinters Methods on Forecasting Additive Seasonal Data

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Abstrak

Nilai persediaan toko dan gudang yang melayani perusahaan ritel dilaporkan pada setiap akhir tahun, khususnya pada tanggal 31 Desember terdapat sekitar 26%, sehingga dapat dikatakan bahwa nilai persediaan toko dan gudang masuk kedalam kategori rendah. Oleh karena itu, dalam menentukan prediksi nilai persediaan yang tersedia untuk dijual, peneliti ingin melakukan peramalan pada data *Advance Retail Sales: Food and Beverage Store*s yang mempunyai pola musiman aditif. Pola musiman aditif adalah pola data musiman yang menunjukkan adanya tren yang relatif konstan seiring waktu. Dalam hal ini, metode yang cocok digunakan adalah *Fourier Series Analysis* dan metode Holt-Winters. Tujuan penelitian adalah untuk mengetahui metode mana yang paling cocok untuk diterapkan pada data tersebut melalui perbandingan hasil dari dua metode tersebut berdasarkan nilai MAPE. Hasil penelitian menunjukkan bahwa metode Holt-Winters cocok untuk data *Advance Retail Sales: Food and Beverage Stores* adalah karena memiliki nilai MAPE yang paling kecil, yaitu sebesar 1,477%.

Kata Kunci : *Forecast, FSA, Holt-Winters, RSDBSN*

Abstract

The value of store and warehouse inventory serving retail companies is reported at the end of each year, specifically on December 31, there is around 26%, so it can be said that the value of store and warehouse inventory is in the low category. Therefore, in determining the prediction of the value of inventory available for sale, researchers want to forecast Advance Retail Sales data: Food and Beverage Stores data which shows additive seasonal pattern. An additive seasonal pattern shows seasonal pattern with relatively constant trend over time. In this case, the suitable methods to use are Fourier Series Analysis and the Holt-Winters method. The aim of the research is to find out which method is most suitable to be applied to this data by comparing the results of the two methods based on the MAPE value. The research results show that the Holt-Winters method is suitable for Advance Retail Sales: Food and Beverage Stores data because it has the smallest MAPE value, namely 1.477%.

Keywords: *Forecast, FSA, Holt-Winters, RSDBSN*

INTRODUCTION

Sales-related services include merchandise sold by companies engaged in retail trade that is paid for by manufacturers or wholesalers and passed on to retailers (Febyani, 2020). Sales are said to be net of refunds and deductions for merchandise returned by customers. Seasonal adjustment estimates are estimates based on current and past data. Therefore, adjustments may become less precise due to changes in economic conditions and other elements that cause significant changes in seasonal patterns and trading days.

Based on first-in, first-out (FIFO) records, merchandise inventories are shown for stores and warehouses serving retail establishments including warehouses that maintain merchandise inventories. It is intended for distribution to retail stores within an organization/company. Most companies report their inventory value at the end of the year for accounting purposes. Based on the data reported at the end of each year, specifically on December 31, there are about 26% of retail inventory estimates from the annual retail trade survey (Louis, advance-retail-sales-food-and-beverage-stores, 2020).

Forecasting is an attempt to predict something that will happen in the future based on past data using a systematic scientific and qualitative method (Makridakis, 1999) and can also determine a certain policy that is influenced by the size of the resulting forecasting value (Ramadhan, 2020). Forecasting in this case is used to anticipate downward economic trends. Then from existing forecasts, economic policy makers can also use predictions for analysis related to economic policy in the future, analyze market trends and determine the direction of the economy (Louis, 2020). Advanced Retail Sales of Food and Beverage Stores is an indicator measured in millions of dollars. As of September 1 and updated monthly,

the latest month is a preliminary estimate based on data from the larger Monthly Retail Trade Company survey (Analytics, 2023). Therefore, in determining the value of inventory available for sale in the future, researchers use forecasting, and this forecasting will be used to determine the appropriate method to use for Advance Retail Sales data: Food and Beverage Stores.

Various kinds of forecasting methods that can be used to make predictions in time series, must look at various aspects, namely the data pattern and the type of model observed, to the desired level of forecasting accuracy. Proper forecasting of Advance Retail Sales data: Food and Beverage Stores is needed (Darmawan, 2018). Data like this can be modeled by involving trigonometric functions, namely sine and cosine. One method that uses trigonometric functions is Fourier Series Analysis (FSA). Fourier Series Analysis (FSA) is seasonal forecasting using a fourier series that decomposes a periodic signal generated from a function into the sum of sine and cosine signals. Then a similar method from the aspect of data patterns is also carried out using the HoltWinters Analysis method. The purpose of the research is to find out which method is best to apply to the data, acomparison will be made of the results of the two methods, namely Fourier Series Analysis and Holt-Winters Analysis using the MAPE value.

RESEARCH METHODS

In this study, an analysis of time series data using the Fourier Series Analysis (FSA) method and the Holt-Winters Analysis method using RStudio software was carried out. The research stages are shown in Figure 1.

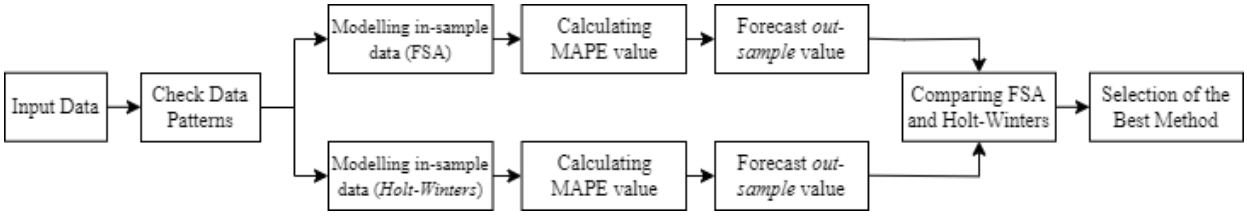

Figure 1. Research Stages

The research stage starts from data input and then checks the data pattern. After the data pattern is checked and there is a seasonal data pattern, it can be continued to the in-sample data modeling stage using the Fourier Series Analysis (FSA) method and the Holt-Winters Analysis method. After modeling, the MAPE value will be calculated using both methods. The next stage is forecasting the out-sample value. After that, the MAPE values of the two methods will be compared to decide which method is best for Advance Retail

Sales data: Food and Beverage Stores.

The next section will explain the supporting theories in analyzing this research, including an explanation of the Fourier Series Analysis (FSA) method and the Holt-Winters Analysis method.

Time Series Analysis

Time Series is data that is used according to time in the form of daily, weekly, monthly and even yearly. In the concept of a series, the time interval between observations must be the same. Time series analysis is an analysis performed based on the error of a variable in the past. The purpose of such analysis is to forecast or predict future time series patterns by implementing patterns found in time series on historical data. Forecasting using time series is based on forecasting the future according to the historical data that has been obtained. Examples of time series data are daily stock data, monthly inflation data, BI interest rate data and so on (Febyani, 2020).

Forecasting

Forecasting is an attempt to predict something that will happen in the future based on past data using a systematic scientific and qualitative method (Makridakis, 1999).

Forecasting consists of two types, namely qualitative forecasting and quantitative forecasting. Qualitative forecasting is forecasting based on the opinion of a party and the data cannot be presented strictly into a number or value. The results of the forecasting made are very dependent on the person who compiled it because the results of the forecasting will be determined based on intuition, opinion and knowledge and experience of the preparation. Second is quantitative forecasting, quantitative forecasting is forecasting that analyzes the variables to be forecasted with time variables so that it can be concluded that quantitative forecasting is more effective than qualitative forecasting.

The forecasting results obtained depend on the method used. The good and bad of the forecasting day is seen from the difference or deviation between the forecasting results and the actual data or original data, in other words, a reality that occurs. If the deviation is smaller between the forecasting results and the actual data, the better the method used. The forecasting period can be categorized into three, which are as follows (Hasbiollah, 2015):

1. Short-term Forecasting

   Forecasting for a period of less than three periods ahead. Examples include planning purchases, workforce numbers, and production levels.

2. Medium-Term Forecasting

   Forecasting that covers monthly to 3 years. Examples such as production budgets, cash, and analyze operating plans.

3. Long-term Forecasting

   Forecasting for more than three periods ahead. Examples include new product and facility development (Agan, 2022).

Time Series Data Patterns

In determining the time series method/model, there is a visual examination of the data pattern (Primandari, 2016). Here are four types of data patterns in time series:

1. Horizontal Pattern

   A horizontal pattern occurs when data fluctuates around a constant value or average forming a horizontal line. Such data is referred to as stationary data. Below is an example of data with a horizontal pattern:

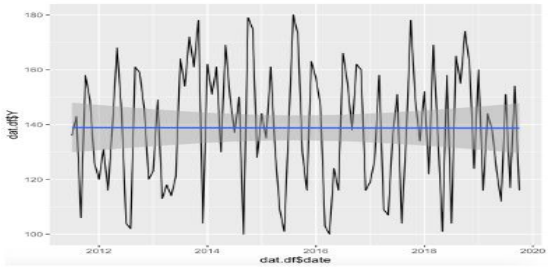

Figure 2. Horizontal Pattern Example

2. Cyclical Pattern

   A cyclical pattern is a time series pattern that shows a cyclical increase or decrease around a trend or normal condition. This up or down pattern is usually found in industrial and financial data because basically the data must experience changes every year and even monthly. The following is an example of data with a cyclical pattern:

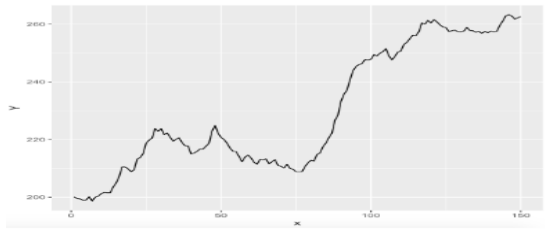

Figure 3. Cyclical Pattern Example

3. Seasonal Pattern

A seasonal pattern is a movement that repeats itself regularly for about a year such as a pattern that repeats weekly, monthly, or quarterly and so on. This pattern looks like peaks and valleys that repeat at the same interval. Quarterly data and monthly data are affected by seasonal factors because of the repeating pattern. While the graph between the data graphs is a trendline. The following is an example of data with seasonal patterns:

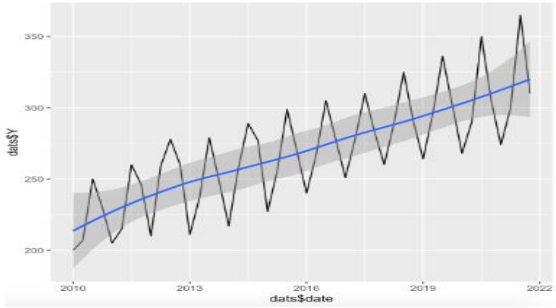

Figure 4. Seasonal Pattern Example

4. Trend Pattern

The pattern appears if the data has an increase and decrease within a certain period of time so that the results of this pol only go in one direction, namely up or down. The following is an example of data that has a trend pattern:

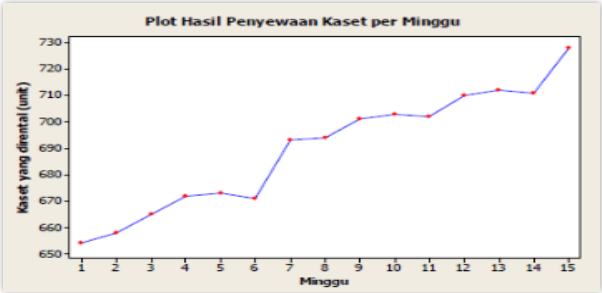

Figure 5. Trend Pattern Example

(Primandari et al., 2016)

Fourier Series Analysis (FSA)

Fourier Series Analysis is a method used to represent a repeating function with a certain pattern as a sum of waves sin and cos with a certain amplitude and frequency. The model used in this research is the Fourier Series Analysis (FSA) equation model. The general equation of the Fourier Series Analysis (FSA) model is as follows (Christienova, 2018):

$$\hat{Y}_t = a_o + b_o t + a_1 cos(\omega t) + b_1 sin(\omega t) + \ldots + a_k cos(k\omega t) + b_k sin(k\omega t) \quad (1)$$

with:

$\hat{Y}_t$: fitted value or forecast at time t,

a_o : a constant used in determining the level of time series data,

b_o : trend estimation based on time series data,

$a_1, b_1, a_2, b_2, \ldots$: coefficients that define amplitude and phase,

ω : $2\pi f/n$ (omega),

k : harmonics of ω.

The FSA equation is usually only up to 5th order because if the order is more than 5 then the equation is not simple (parsimony). Here are the analysis steps using R Studio software (Sasmita & Darmawan, 2017):

1. Determining the monthly trend by dividing the annual average by the seasonal length (S) can be seen in the following equation.
$$Mean(Y_t - Y_{t-12}) \quad (2)$$
2. Perform trend centering of the mean Y_t and average over time t, then determine the trend line equation.
3. Determine the trend values for the initial data and the last data, t=1 and t=N.
4. Determine the deviation value of the trend by subtracting the trend projection from each actual data when generating a new data series centered at zero (0)
5. Input the detrended series and trigonometric values to the multiple regression values for the detrended actual values $cos(\omega_t), sin(\omega_t), cos(2\omega_t), sin(2\omega_t), \ldots$
6. Fit the FSA (Fourier Series Analysis) model to the detrended values using the model from multiple regression to obtain the coefficients. $a_1\, b_1$, $a_2 b_2$, … in minimizing the sum of squares of the error.
7. Calculate the amplitude at each frequency. Finding the amplitude value can use the following equation.
$$A_i = \sqrt{(a_i^2 + b_i^2)} \quad (3)$$
8. Discard the frequencies in the value of $A_i > 0.05$. If the frequency values at 1st and 2nd order produce amplitude values > 0.05 and the frequency at 3rd order has amplitude < 0.05, then the data has similarities to spectral regression with 2nd order.
9. Perform forecasting of the out-sample values according to the order set at step 8 by showing the results based on the trend and seasonal components.

Holt-Winters *Analysis*

The Holt-Winter forecasting method is based on three smoothing equations, namely

the level smoothing equation, the trend component, and the seasonal component, where each smoothing equation has different parameters. The Holt-Winter method has two types of seasonality, namely additive seasonality and multiplicative seasonality. Additive Holt-Winter is used when the data shows a trend and seasonal fluctuations that are relatively constant over time. Meanwhile, the multiplicative Holt-Winter method is used when the data shows a trend and seasonal fluctuations that increase over time (Andriani, Wahyuningsih, & Meiliyani, 202).

The equation for the additive Holt-Winter method can be seen in the following equation:

$$L_t = \alpha(X_t - S_{t-s}) + (1 - a)(L_{t-1} + T_{t-1}) \quad (4)$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1} \quad (5)$$

$$S_t = \gamma(X_t - L_t) + (1 - \gamma)S_{t-s} \quad (6)$$

$$F_{t+m} = L_t + T_t m + S_{t-s+m} \quad (7)$$

The level, trend, and seasonal components of the additive Holt-Winter method can be initialized using the following equation.

$$L_s = \frac{1}{s}(X_1 + X_2 + \ldots + X_L) \quad (8)$$

$$T_s = \frac{1}{s}\left(\frac{X_{s+1} - X_1}{s} + \frac{X_{s+2} - X_2}{s} + \ldots + \frac{X_{s+s} - X_s}{s}\right) \quad (9)$$

$$S_i = X_i - L_s \quad (10)$$

The smoothing equation for the multiplicative Holt-Winter method is as follows.

$$L_t = \alpha \frac{X_t}{S_{t-s}} + (1 - \alpha)(L_{t-1} + T_{t-1}) \quad (11)$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1} \quad (12)$$

$$S_t = \gamma \frac{X_t}{L_t} + (1 - \gamma)S_{t-s} \quad (13)$$

$$F_{t+m} = (L_t + T_t m)S_{t-s+m} \quad (14)$$

Level, trend, and seasonal components in the Holt-Winter multiplicative method using initialization using the following equation (Andriani, Wahyuningsih, & Meiliyani, 2022).

$$L_s = \frac{1}{s}(X_1 + X_2 + \ldots + X_L) \quad (15)$$

$$T_s = \frac{1}{s}\left(\frac{X_{s+1} - X_1}{s} + \frac{X_{s+2} - X_2}{s} + \ldots + \frac{X_{s+s} - X_s}{s}\right) \quad (16)$$

$$S_i = \frac{X_i}{L_s} \quad (17)$$

where:

α : level smoothing parameter, $0 \leq \alpha \leq 1$,

β : trend smoothing parameter, $0 \leq \beta \leq 1$,

γ : seasonal smoothing parameters, $0 \leq \gamma \leq 1$,

X_t : actual data at time t,,

L_t : level smoothing at time t,

T_t : trend smoothing at time t,

S_t : seasonal smoothing at time t,

s : seasonal length,

L_s : level smoothing at time s,

T_s : trend smoothing at time s,

S_i : seasonal smoothing at time i, $i = 1,2,\ldots,s$,

F_{t+m} : forecasting at time $(t+m)$.

Calculating Mean Absolute Percentage Error (MAPE)

The MAPE measurement aims to measure the absolute average percentage of the forecast error. Several opinions and based on research conducted by previous researchers, namely Nurkhasanah et al. (2015) the model is said to have good performance if the MAPE error value is below 10%. The MAPE calculation uses the following formula:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{X_i - \hat{X}_i}{X_i}\right| x\ 100\% \quad (18)$$

where,

n : Amount of data;

X_i: Actual data;

$\hat{X}_i$: Presumtive value.

The following is the accuracy using the MAPE value can be seen in the following table (Noviani, 2020).

Table 1 MAPE Accuracy Criteria

MAPE Value	Accuracy Criteria
< 10%	Very Good
10% - 20%	Good
21% - 50%	Simply
>50%	Bad

The table provides a guideline for interpreting the accuracy of forecasting methods based on the MAPE value. It allows researchers to assess the performance of different forecasting methods and determine their level of accuracy.

RESULTS AND DISCUSSION

This study uses secondary data, namely Advance Retail Sales data: Food and Beverage Stores. The data is data on food and beverage retail sales in America. The study uses monthly data from January 2008 to August 2023. This data is obtained from the FRED website. Before any further analysis is carried out, information regarding the characteristics of the data is essential as material relating to the application of methods and according to the characteristics of the data. Before further analysis is carried out, information on the characteristics of the data is essential as it relates to the application of methods and is appropriate to the characteristics of the data. To describe the characteristics of the data, data exploration is done descriptively through chronological progression graphs and summary statistics of the data and time series components. Figure 6 shows the data development of Advance Retail Sales: Food and Beverage Stores from January 2008 to August 2023.

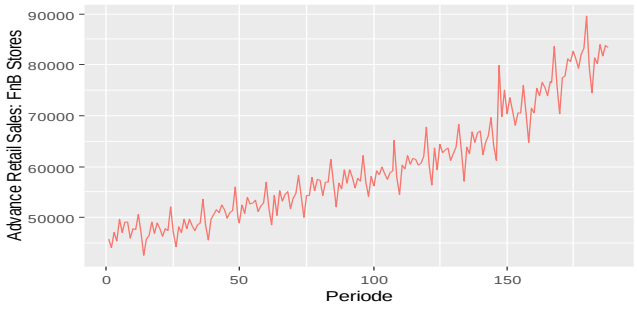

Figure 6. Time Series Plot of Advance Retail Sales Data: Food and Beverage Stores

The figure shows a time series plot of advance retail sales data for food and beverage stores in Indonesia from January 2008 to November 2023. The data is measured in billions of Indonesian rupiah. The plot shows that average retail sales for food and beverage stores have been on a general upward trend over the past 15 years. There have been some fluctuations along the way, but the overall trend has been positive.

Figure 2 is a time series plot of the data which shows that the Advance Retail Sales: Food and Beverage Stores has a seasonal pattern that contains a trend and is additive. In this study, the data is divided into 185 insample data and 3 out-sample data.

Fourier Series Analysis (FSA)

By using the steps listed in the research method, significant amplitude values are obtained at orders 2 and 5, so this study will use FSA of orders 2 and 5. The MAPE value for h = 2 and h = 5 is 4.756%.

With values:

$a_o = 41201.346$ $a_2 = 1571.084$ $a_5 = 1243.494$
$b_o = 196.8979$ $b_2 = -826.0995$ $b_5 = 1220.2488$

so that the following equation is obtained:

$$\hat{Y}_t = 41201.346 + 196.8979t + 1571.084(2\omega t) - 826.099(2\omega t) + 11243.494(5\omega t) - 1220.248(5\omega t)$$

From the equations that have been listed, a forecast is made for the in-sample data which is then compared with the actual data. The comparison between the predicted data and the actual data can be seen in Figure 3. The figure shows that the forecast results have a trend pattern similar to the trend pattern in the actual data. However, the seasonal pattern in the forecast results does not follow the actual data pattern completely.

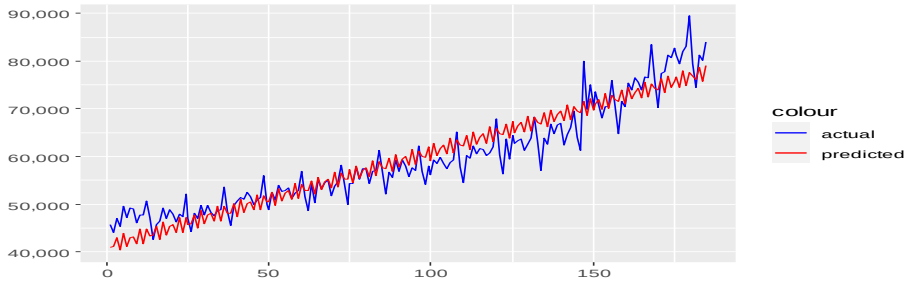

Figure 7. Plot of Actual and Predicted Data of Advance Retail Sales: Food and Beverage Stores Using the FSA Method

Furthermore, the forecasting results are evaluated both on in-sample data and on out-sample data. The evaluation results of the FSA model obtained are shown in Table 2 below.

Table 2. Comparison of MAPE values on in-sample and out-sample data using FSA

MAPE	
in-sample	out-sample
4.756%	5.992%

The MAPE values for in-sample and out-sample data are 4.756% and 5.992% respectively. From the comparison of the MAPE value on in-sample and out-sample data, it is obtained that the best MAPE value is found in in-sample data, namely 4.756%. These results suggest that the FSA method may be more suitable for forecasting the Advance Retail Sales data: Food and Beverage Stores within the observed time period.

Holt-Winters Analysis

Based on the calculation results, the values of α, β, γ are calculated through initial value smoothing, trend smoothing, and seasonal smoothing. From the analysis results, the values of $\alpha = 0.428, \beta = 0.216$, dan $\gamma = 0.381$. The model with these smoothing coefficients produces the forecast shown in Figure 8.

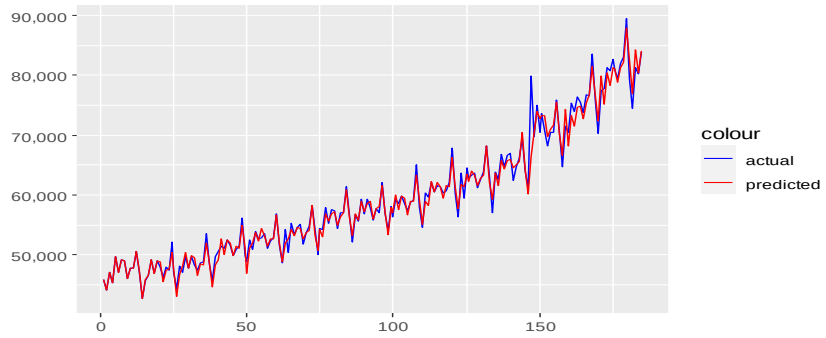

Figure 8. Plot of Actual and Predicted Data for Advance Retail Sales: Food and Beverage Stores Using the Holt-Winters Method

It can be seen that the forecasting results using the Holt-Winters method produce values that follow the actual data pattern very well. The following are the results of calculations using Holt-Winters Analysis:

Table 3. Holt-Winters Analysis calculation results

MAPE	
in-sample	out-sample
1.477%	0.459%

The value in the MAPE in sample column is 1.477%, indicating that the Holt-Winters Analysis method has a MAPE of 1.477% for the Advance Retail Sales data: Food and Beverage Stores, while the MAPE value in the out sample column is 0.459%. This indicates that forecasting on testing data is more accurate.

Forecasting Results

The forecasting process on Advance Retail Sales data: Food and Beverage Stores is done to estimate a value in the future based on past data. Comparison of the forecasting value with the actual monthly value of Advance Retail Sales: Food and Beverage Stores for

three periods can be seen in the following table.

Table 4. Comparison of forecasting values with actual data

Period	Actual Data	FSA Forecasting Value	Holt-Winters Forecasting Value
1 Juni 2023	81796	76916.74	81882.84
1 Juli 2023	83698	78061.40	83609.54
1 Agustus 2023	83460	79056.28	82486.44

Based on Table 3, it can be seen that the forecasting value for the next three periods on Advance Retail Sales data: Food and Beverage Stores using Holt-Winters follows the actual data more closely than the forecasting results in Fourier Series Analysis (FSA). The plot of forecasting values obtained using the Holt-Winters method can be seen in Figure 9.

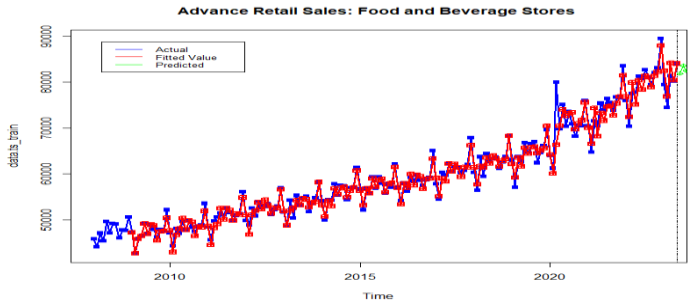

Figure 9. Plot of Actual, Fitted, and Forecasted Advance Retail Sales: Food and Beverage Stores

Based on Figure 9, it can be seen that the fitted value is able to follow the pattern of the actual data, and the resulting forecasting value follows the pattern in the actual data of the previous period.

Selection of the Best Method

After analyzing using FSA and Holt-Winters Analysis, the best method will be selected by comparing the MAPE values of the two methods used.

Table 5. Comparison of MAPE values in FSA and Holt-Winters Analysis

Fourier Series Analysis		Holt-Winters Analysis	
in-sample	out-sample	in-sample	out-sample
4.756%	5.992%	1.477%	0.459%

Table 5 comparing the Mean Absolute Percentage Error (MAPE) values for two methods: Fourier Series Analysis (FSA) and Holt-Winters Analysis. Based on Table 5. obtained MAPE value with the FSA method is 4.756% in-sample and 5.992% in out-sample, while the MAPE value with the Holt-Winters method is 1.477% in in-sample and 0.459% in out-sample. The criteria for selecting the best method is by comparing the smallest MAPE value in both methods. From the calculation results it can be concluded that the best method that can be used on Advance Retail Sales data: Food and Beverage Stores is the Holt-Winters method with a MAPE value of 1.477%.

CONCLUSION

From the calculation results using the FSA method, the following equation is obtained:

$$\hat{Y}_t = 41201.346 + 196.8979t + 1571.084(2\omega t) - 826.099(2\omega t) + 11243.494(5\omega t) - 1220.248(5\omega t)$$

Based on the results of calculations with FSA and the Holt-Winters method, it is found that the best method used to analyze Advance Retail Sales data for Food and Beverage Stores is the Holt-Winters method. This is based on the comparison of MAPE values for both in-sample and out-sample in the FSA method and the Holt-Winters method. By using FSA, the MAPE value = 4.756% in the in-sample and the MAPE value = 5.992% in the out-sample are obtained. While using the Holt-Winters method obtained MAPE value = 1.477% in in-sample and obtained MAPE value = 0.459% in out-sample.

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